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IN THE NAME OF

Robert J. Collins

OF

Real Sport, Inc.

FOR

# DETECTING MOVEMENT CHARACTERISTICS OF AN OBJECT

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*Prepared by*

Merle W. Richman, III

Express Mail No. EJ757527575US

# DETECTING MOVEMENT CHARACTERISTICS OF AN OBJECT

## BACKGROUND OF THE INVENTION

### 1. *Field of the Invention*

This invention relates to systems and methods of detecting the movement characteristics of an object, in particular, a sports object such as a baseball, golf ball or other sports object.

### 2. *Description of Related Art*

It is desirable to be able to predict the trajectory and ultimate movement path of an object by measuring characteristics of the object at some point in time. This is particularly true for sports objects. For example, at a driving range with a limited movement area or many golfers, a golfer striking a golf ball would like to know the projected full movement of a struck golf ball. By measuring movement characteristics of the ball, some prior art systems attempt to determine the projected movement characteristics of the ball, i.e., distance traveled in air and direction. Some prior art systems modify the ball in a way that is obvious and potentially distracting to the user.

In addition, prior art systems are large and complex. A need exists for a system that may be used by an individual in an enclosed environment, such a backyard or garage that is accurate and affordable. Such a system should also be extendable to other struck objects such as a baseball at a batting cage. The present invention provides such a system and method.

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## SUMMARY OF THE INVENTION

The present invention is system that measures and displays information about the movement of an object, in particular a sports object such as a golf ball. In particular, the device measures the speed and direction of movement, as well as spin rate and spin axis orientation of the object.

A method and article of manufacture of the invention for determining a movement characteristic of an object includes reflecting electro-magnetic energy from a sensor off the object. The electro-magnetic energy reflected off the object at the sensor is then received and parameters of a model of the movement of the object determined based on the reflected electro-magnetic energy. Then a movement characteristic of the object based on the determined model parameters is determined

The movement characteristic of the object that the method may determine includes the speed, distance, location, spin angle, or spin rate. In a preferred embodiment, electro-magnetic energy from three sensors may be reflected off the object and the electro-magnetic energy reflected off the object may be received at the three sensors. Further, each sensor's electro-magnetic energy transmission path may be non-parallel to the movement path of the object.

The sensor may be a Doppler radar sensor or a continuous wave Doppler radar sensor. In a further embodiment, the electro-magnetic energy from the sensors may be reflected off a contrast marker of the object and the electro-magnetic energy reflected off the contrast marker of the object may be received at the sensors. The contrast marker of the object may be highly reflective of the electro-magnetic energy generated by the sensors.

Sub. a27 An apparatus of the invention for determining a movement characteristic may include an object having a movement path and an electro-magnetic sensor. The sensor generates electro-magnetic energy to be reflected off the object and receives the electro-magnetic energy reflected off the object. The apparatus also includes means for determining parameters of a model of the movement of the object based on the reflected electro-magnetic energy and means for determining a movement characteristic of the object based on the determined model parameters

The means for determining a movement characteristic may includes means for determining one of the speed, direction, location, spin angle, and spin rate of the object based on the determined model parameters. The apparatus may also include a second and a third electro-magnetic sensor. The second sensor generates electro-magnetic energy to be reflected off the object and receives the electro-magnetic energy reflected off the object. The third sensor also generates electro-magnetic energy to be reflected off the object and receives the electro-magnetic energy reflected off the object. In the apparatus, the sensor's electro-magnetic energy transmission path may be non-parallel to the movement path of the object.

The sensor of the apparatus may be a Doppler radar sensor or a continuous wave Doppler radar sensor. Further, the object of the apparatus may include a contrast portion or contrast marker. In this apparatus, the sensor generates electro-magnetic energy to be reflected off the contrast marker of the object and receives the electro-magnetic energy reflected off the contrast marker of the object.

## BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a side view diagram of an exemplary system of the present invention being used to determine characteristics a golf ball struck by a golfer.

FIG. 2 is a top view diagram of the exemplary system of the present invention shown in FIG. 1.

FIG. 3 is a diagram of sensor arrangement enclosure for detecting movement characteristics of a golf ball according to the present invention.

FIG. 4 is a block diagram of a sensor system according to an embodiment of the present invention.

FIG. 5 is a flowchart of a method for determining movement characteristics of an object based on received sensor data according to the present invention.

FIG. 6 is a plot of a digitized Doppler radar signal generated by a golf ball passing a sensor at approximately 49 m/s (109 mi/hr) with its closest point of approach to the sensor at 82.9 milliseconds at a distance of 0.729 m (28.7 in).

FIG. 7 is a plot of the frequency domain representation of the digitized Doppler radar signal shown in FIG. 6 where the vertical or frequency axis has been converted to velocity.

FIG. 8 is a flowchart of a method for determining the parameters of a speed/distance model according to the present invention.

FIG. 9 is a diagram of the basic geometry that exists between an object at point A passing a radar sensor 110.

FIG. 10 is a flowchart of a method for determining the spin rate and angle of a target object according to the present invention.

FIG. 11A is a side cut away view diagram of golf ball with two contrasting regions according to the present invention.

FIG. 11B is a top cut away view diagram of the golf ball shown in FIG. 11A.

FIG. 12A is a side cut away view diagram of golf ball with a single contrasting region according to the present invention.

FIG. 12B is a top cut away view diagram of the golf ball shown in FIG. 12A.

FIG. 13 is a cut away view diagram of golf ball with a four contrasting regions arranged in a tetrahedron configuration according to the present invention.

FIG. 14A is a front cut away view diagram of golf ball with two contrasting regions where the size of the contrasting regions is different according to the present invention.

FIG. 14B is a side cut away view diagram of the golf ball shown in FIG. 14A.

FIG. 15 is a flowchart of a method for identifying the times contrasting marks of a target object appear in a radar beam according to the present invention.

FIG. 16 is a flowchart of an exemplary method for determining a spin angle from amplitude peak measurements.

FIG. 17 is a diagram of the geometry associated with determining a spin angle based on two amplitude peak measurements.

Like reference numbers and designations in the various drawings indicate like elements.

## DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

Throughout this description, the preferred embodiment and examples shown should be considered as exemplars, rather than as limitations on the present invention.

FIG. 1 is a side view diagram and FIG. 2 is a top view of an exemplary system 10 according to the present invention. The system 10 measures and displays information about the movement of an object, in this case golf ball 42 struck by a user 44. In particular, the system 10 determines the speed and direction of movement 40 of the golf ball 42, as well as the spin rate and spin axis orientation of the ball 42. The system includes a sensor array enclosure 20 and personal computer 118 coupled by a cable 22. In this embodiment, a user 44 strikes a golf ball 42 from a mat 60. When the ball passes in the field of view of the sensor array 20, the sensor array generates signals transmitted to the personal computer 118 via cable 22. The personal computer 118 determines the speed and direction of movement 40 of the golf ball 42, as well as spin rate and spin axis orientation of the ball 42 from the sensor array 20 signals.

As shown in FIG. 1 and FIG. 2, the sensor array 20 has a vertical field of view 52 and a horizontal field of view 50. The sensor array's 20 field of view varies as a function of the type of sensors, numbers of sensors, and position of sensors in the sensor array 20. An exemplary sensor array 20 is shown in FIG. 3. The sensor array 20 includes three radar front-end sensors 110, 112, and 114, a radar signal processor 30, and a signal transmission cable 22. In an exemplary embodiment, each front-end sensor 110, 112, 114 is a continuous wave ("CW") Doppler radar. Using the data generated by each sensor 110, 112, and 114, the invention can determine the range

to the object at every point in time and the actual speed of the object, regardless of the direction of movement through the sensor's field of view.

The signal processor 30 in the sensor enclosure 20 stores the stream of digitized samples in a circular buffer in local memory (RAM 120 as shown in FIG. 4). As it stores each triplet of samples (one from each sensor 110, 112, and 114), the processor 30 examines the signal values. When all three sensors generate a sample value above 200 (out of maximum 256) within 256 samples of each other, a trigger is registered. When a trigger is registered, the processor 30 gathers the previous 50 milliseconds of data, records an additional 150 milliseconds worth of data, and transmits these 200 milliseconds of data to the personal computer 118 for further processing.

When the personal computer 118 receives the 200 milliseconds of data from the three sensors, it proceeds with the signal processing as described below. Once the system 10 determines the launch parameters, in particular the ball's location, speed, direction, spin rate, and spin axis orientation, the system 10 may also 1) communicate the computed values on the personal computer 118 with text, graphics, or audio; 2) store the computed values in a database, along with information about the golfer 44 and the club used; 3) compute the projected trajectory of the ball 42 including bounces and roll; 4) display the projected movement graphically as well as summary statistics, such as distance traveled to the first bounce (carry distance), distance from the target line at the first bounce (carry dispersion), distance traveled to the final resting place (total distance), and distance from the target line at the final resting place (total dispersion), peak height; and 5) compute and display statistics (e.g., averages) of the computed values (launch

parameters as well as values derived from the trajectory prediction) from a series of shots.

FIG. 4 is a block diagram of an exemplary radar signal processor 30 according to the present invention. The radar signal processor 30 includes an amplifier 111, an Analog-to-Digital "A/D" converter/processor 116, and a random access memory ("RAM") 120. The radar system processor 30 is coupled to the radar front-end sensors 110 (112 and 114 as shown in FIG. 3). Each radar front-end sensor 110, 112, and 114 feeds a signal to the amplifier 111. The amplifier 111 increases the signal strength of the radar sensor signals for conversion from an analog signal to a digital signal by the A/D converter/processor 116. The A/D converter/processor 116 converts the analog signal to a digital signal by sampling the signal at a fixed rate and converting the analog samples to digital samples and stores the data in the RAM 120 for batch processing by the personal computer ("PC") 118.

In an exemplary embodiment, each radar sensor 110, 112, and 114 is a commonly available K-band (24.125 GHz) Doppler radar front end with radar antenna. In another embodiment, each sensor is a commonly available X-band (10.5 GHz) Doppler radar front end with radar antenna. In a further embodiment, each sensor is a commonly available Ka-band Doppler radar front end with radar antenna. For simplicity, the description below refers only to the K-band sensor embodiment although it is understood that an X-band or Ka-band sensor could also be employed. The signal generated by each Doppler sensor is a variable voltage frequency signal where the voltage frequency reflects the velocity of an object moving towards or away from the sensor. The radar antenna for each sensor has field of view of about 120 degrees both horizontally and vertically. The amplifier 111 is a product of Orion Engineering of Clearwater, Florida. The

amplifier 111 includes a filter to remove low frequency (less than 500 Hz) and high frequency (greater than 15 KHz) components from each radar sensor signal.

The A/D converter/processor 116 is a product of Summit Engineering of Encinitas, California. The processor 116 communicates with the PC 118 using a universal serial bus ("USB") protocol. The processor 116 samples the analog sensor signal at a fixed rate of 28,000 Hz and generates an 8-bit digital sample for each analog sample. The digital samples may be initially stored in the RAM 120 prior to transmission to the PC 118. The PC 118 may be any commercially available PC that includes a USB interface.

FIG. 5 is a flowchart of the process 200 performed by the radar processor 30 and PC 118 to convert the analog Doppler signal from each sensor 110, 112, and 114 to speed/distance data about an object 42 passing the sensors. In step 202, the radar signal is converted to a digital signal. The radar signal processor 30 performs this step. The digital data representing the analog Doppler signal is transmitted to the PC 118 via a USB. FIG. 6 is a plot of a digitized Doppler radar signal generating by a golf ball passing a sensor at approximately 49 m/s (109 mi/hr) with its closest point of approach to the sensor at 82.9 milliseconds at a distance of 0.729 m (28.7 in). As can be seen from this plot at approximately 65 milliseconds, a ball enters the radar beam of a sensor 110, 112, or 114, and the amplitude of the Doppler radar sensor signal starts increasing. The Doppler radar sensor signal amplitude continues to rise and the frequency of the signal decreases until the approximate time point of 81 milliseconds. The amplitude increases because the ball is moving into the central part of the sensor beam and is getting closer to the radar sensor. The sensor signal frequency decreases because of the increasing cosine error of the

sensor, in particular the velocity of the ball relative to the radar appears to be decreasing as the ball approaches and crosses the radar beam.

Between 82 milliseconds and 84 milliseconds, the signal generated by the golf ball 42 is nearly absent. At this time point the ball is moving past the radar sensor with minimal movement either toward or away from the radar sensor. The frequency and the amplitude of the signal at this time are approximately zero. After the ball 42 passes the sensor 110, 112, and 114 (between time points 84 milliseconds and approximately 142 milliseconds) the amplitude decreases and the frequency increases. The radar signal amplitude decreases as the ball 42 moves away from the radar sensor 110 and the frequency increases as the cosine error decreases, *i.e.*, the angle between the ball's 42 velocity vector and a vector from the radar sensor 110 to the ball 42 decreases. In this plot, the radar signal also includes temporary increases in the amplitude in the time period from 100 to 120 milliseconds and after 140 milliseconds due to a club head (which struck the ball 42) entering the field of view. In this plot at approximately 142 milliseconds, the ball strikes a net (not shown). The present invention uses the characteristics of Doppler signal generated by ball 42 passing transverse to a sensor, to determine the speed/distance model for the ball in step 208.

At step 204 (FIG. 5) the time domain, digital data representing the Doppler signal from each radar sensor is transformed to its corresponding frequency domain signal. In one embodiment, the PC 118 performs discrete Fourier transforms at regular time intervals across the voltage or time domain digital Doppler data to generate the corresponding frequency domain signal data. An exemplary FFT algorithm is described by Press, William H., et al. *Numerical Recipes in C.*

Cambridge University Press. 1992, which is incorporated by reference for its teachings on FFT algorithms. A short time period or small number of digital samples are used to generate each Fourier transform so the frequency components at any given instant may be determined.

In detail, the preferred embodiment uses a Fast Fourier Transform ("FFT") to transform the time domain Doppler signal into a set of cosine components that when added together represent the untransformed Doppler signal. A complex FFT generates a phase and an amplitude for each cosine component. In the preferred embodiment, only the amplitude is needed so a real FFT may be used to determine the cosine components in each set of Doppler signal digital samples. In one embodiment, the PC 118 applies a Blackman window to every 150 Doppler digital samples (approximately five milliseconds) and then zero-pads the windowed data to yield 512 digital samples. The resultant 512 digital time domain samples are converted to frequency domain samples using an FFT.

FIG. 7 is a plot of the frequency domain signal generated from the digital data signal shown in FIG. 6. In FIG. 7, high amplitude frequency components are darker. The prominent V-shaped response represents the frequency response of the radar sensor 110 as the golf ball passes the sensor. The additional high amplitude (dark) responses beginning around 100 milliseconds represent the frequency response of the radar sensor 110 as the golf club and perhaps part of the golfer pass the sensor. The steep line that is evident above the departing ball is a harmonic of the ball signal. In Fig. 7, the vertical axis represents the velocity of the ball where the frequency data has been converted to velocity data. The frequency data is converted to velocity data at step 206.

In particular, the data is scaled from frequency to velocity by applying the well-known Doppler equation. The Doppler equation is $v = \frac{cf_D}{2f_C}$ where $v$ is the velocity in units of $\frac{m}{s}$ that corresponds to the measured Doppler shift frequency $f_D$ in units of Hertz (as reported by the radar sensor), $c$ is the speed of light in units of $\frac{m}{s}$ and $f_C$ is the radar carrier frequency in units of Hertz. For the K-band sensor we are using, $f_C = 24{,}125{,}000{,}000$ Hertz. Accordingly, the frequency data can be scaled to the velocity data inherent in the Doppler radar signal.

In the next step 208, the converted velocity digital data is used to derive a speed and distance model for the object 42 for each sensor. In one embodiment, a speed/distance equation is employed, the speed/distance equation is: $s_t = F_t(S,R,T_R)$ where $s_t$ is the speed that the sensor observes at time $t$. As shown in this equation, the parameters of the speed/distance model $F$ include the target or object speed $S$, the time of closest approach to the sensor $T_R$, and the distance of closest approach to the sensor $R$. In this embodiment, $F$ is defined as

$F(t) = \left| \frac{(t-T_R)S}{\sqrt{(t-T_R)^2 S^2 + R^2}} \right|$. The derivation of this equation is described below.

FIG. 8 is a flowchart of an exemplary method of determining the speed/distance model parameters that provide the best fit to the measured data. The quality of fit of the measured data to a proposed set of speed/distance model parameters is determined by computing the sum of the measured amplitude at each time $t$ and velocity $F(t)$: As shown in FIG. 8 at step 210, the method first estimates the object's speed parameter $S$. In this embodiment, the sum of the

amplitudes at each velocity is computed. The highest velocity that has a sum above background noise present in the signal is selected as an estimate of the ball's true speed $S$.

Parameters $T_R$ and $R$ are estimated at step 212. In particular at every one millisecond interval, $T_R$ is set to the current time. In addition, $R$ is set to each of ten evenly spaced intervals from 0.01 meters to 1.6 meters. For each combination of these parameters, their quality of fit to the measured data is determined. Those combinations of parameters having the best quality of fit are selected as the initial estimate of parameters $T_R$ and $R$.

At step 214, the initial estimate of the parameter $T_R$ is refined. In particular, the method evaluates the quality of fit of the parameter to the data at 0.1 millisecond intervals for the time one millisecond before and after the initial estimate of the parameter $T_R$. The estimate of the parameter $T_R$ that fits the data most closely is selected as the refined estimate of the parameter.

At step 216, the initial estimate of the parameter $R$ is refined. In particular, the method evaluates the quality of fit of the parameter to the data at 0.03-meter intervals over the range from 0.01 meters to 1.6 meters using the refined estimate of the parameter $R$. The estimate of the parameter $R$ that fits the data most closely is selected as the refined estimate of the parameter.

At step 218, a final refinement of the parameters $S$, $T_R$ and $R$ is simultaneously performed. In particular, these parameters are simultaneously refined by applying a *downhill simplex optimization method in multidimensions* as described by Press, William H., and et al. *Numerical*

*Recipes in C.* Cambridge University Press. 1992, which is incorporated by reference for its teachings on downhill simplex optimization methods in multidimensions. The application of this optimization method to the current estimates of the parameters will further refine or adjust the parameters to values that optimize the quality of the fit to the measured data for all the parameters simultaneously.

Above, $F$ was defined as $F(t) = \left| \frac{(t - T_R)S}{\sqrt{(t - T_R)^2 S^2 + R^2}} \right|$. This definition is explained with reference to FIG. 9. FIG. 9 is a diagram of the basic geometry that exists between an object at point A passing a radar sensor 110. As shown in this figure, a right triangle $ABC$ can be defined where $B$ is at the location of the radar, $C$ is the location of the target (or object) at its closest distance to the radar and $A$ is the current position of the target at time $t$. In this triangle, the lengths of the sides are $a$, $b$ and $c$. By definition $a = R$, *i.e.*, the minimum distance between the target and the radar. Further, the time at which the target is at location $C$ is by definition $T_R$.

Given that the target is moving along its path at speed $S$, the time required to travel between points $C$ and $A$ is $t - T_R$. Accordingly, $b = |(t - T_R)S|$. By applying Pythagorean theorem to this result, it is noted $c = \sqrt{(t - T_R)^2 S^2 + R^2}$. Further when the target is at position $A$ at time $t$, the radar measures an apparent speed $s_t$. The apparent speed is determined by the cosine error of the radar and thus is $s = S \cos \alpha$ where $\alpha$ is the angle between the target's direction and the direction from the target to the radar. In this case, $\alpha$ is the angle at vertex $A$ of the right triangle

Sub. a3> $ABC$. Given that the cosine function is defined as $\cos A = \frac{b}{c}$ for a right triangle, accordingly, it follows that $s_t = \left| \frac{(t - T_R)S}{\sqrt{(t - T_R)^2 S^2 + R^2}} \right|$.

The speed/distance model assumed a constant actual target speed $S$. The speed of the object is presumed to be constant or changing slowly and the observation time of the sensor of the object is short enough that a change in speed is not evident for the application shown in FIG 1 and FIG. 2. The speed/distance can be modified to support varying speeds or velocities. For example, the speed/distance model may include another parameter such as constant acceleration, $i.e., s_t = F_t(S, A, R, T_R)$ where $A$ is the acceleration constant. Using a method similar to the method shown in FIG. 8, the parameters $A$, $S$, $R$, and $T_R$ can be estimated by applying them to the actual data and refined by using the optimization method.

In FIG. 3, it was shown that the present invention employs three radar sensors 110, 112, and 114 to determine a three dimensional (3-D) velocity vector for the object 42. The speed/distance model for a single sensor used the function $F(S, R, T_R)$. The speed/distance function can be extended to a system that employs multiple sensors. For a system employing three sensors, the speed/distance function is defined as $(s_{0t}, s_{1t}, s_{2t}) = F_t(S, R_0, T_{R_0}, R_1, T_{R_1}, R_2, T_{R_2})$ where $s_{it}$ is the speed that the sensor $i$ observes at time $t$; $S$ is the actual speed of the target; $R_i$ is the minimum distance between the target and sensor $i$ for all values time $t$; and $T_{R_i}$ is the time at which the target is at the minimum distance $R$ from sensor $i$.

Using the three-sensor speed/distance model, the system 10 (FIG. 1 and FIG. 2) can compute the 3-space movement direction of the target object 42 by combining the range measurements from the individual sensors. In particular, the system 10 determines and displays a linear (in 3-space) direction vector and the constant speed for the object. In addition, the system 10 can determine non-linear curves for the speed and direction in 3-space by revising the speed/distance model. Based on the path observed by the radar sensor, the system 10 can also extrapolate the path of the object before or after the period of observation. In the case of a golf ball, the system 10 can determine the movement characteristics of the golf ball 42 after a golfer strikes it. The system 10 can extrapolate the observed speed and direction back to the initial moment (and position) of the ball's movement. The system can then determine the initial conditions of the golf ball movement, rather than the conditions after the ball has moved several feet through the air.

In FIG. 3, a coordinate system 32 is shown, this coordinate system 32 applicable in this case to measurement of the movement of a golf ball. This system 32 is used to provide a frame of reference for any direction vectors that the system 10 may determine. We must reference the measured direction information to a coordinate system. When a golf ball is struck, there is a preferred direction of movement of the golf ball, known as the target line. The target line is the direction the golfer 44 is attempting to hit the ball. In this embodiment, the system 10 uses a three-dimensional (x, y, z) rectangular coordinate system. In this coordinate system, the z-axis is parallel to the acceleration vector of Earth's gravity, with the negative z-axis pointing in the direction of the gravitational vector (straight down). The x-y plane is thereby, parallel to the nominal ground surface. The positive x-axis is parallel to the target line. The positive y-axis extends to the right of the target line.

As shown in FIG. 3, the coordinate system 32 has a particular location relative to the sensor enclosure. In particular, the origin of the coordinate system 32 is at the edge of the rectangular enclosure that lies on the target line and is closest to the golfer. The placement of the enclosure determines the orientation of the coordinate system (e.g., ground (x-y) plane, target line, etc.). For example, the enclosure will typically have a target-line (x-axis) indicator on it to align the ideal target line. The enclosure is also ideally leveled with the ground. In the system 10, each sensor is positioned and oriented within the enclosure so that all three sensors 110, 112, and 114 have a clear view of the movement path of interest. For measuring the movement of golf balls 42, the sensors are placed at meter coordinates $(0.143, 0.302, 0.024)$, $(0.652, -0.533, 0.024)$ and $(0.624, 0.510, 0.024)$ relative to coordinate system 32. In addition, the center of each radar beam is oriented to point upward at a 45-degree angle and across the target line at a 45-degree angle. The system 10 considers the position of each sensor relative to the arbitrary coordinate system origin is an input parameter when computing the desired parameters of the speed/distance model. Using the above coordinate system, 3-space model and data shown in FIG. 1, the method of FIG. 8 computes the speed/distance parameter values shown in Table 1.

| **Parameter** | **Value** | **Units** |
|---|---|---|
| $S$ | 49.005 | $\frac{m}{s}$ |
| $R_0$ | 0.729 | $m$ |
| $T_{R_0}$ | 0.8285 | $s$ |
| $R_1$ | 0.872 | $m$ |
| $T_{R_1}$ | 0.9365 | $s$ |
| $R_2$ | 0.998 | $m$ |
| $T_{R_2}$ | 0.9203 | $s$ |

**TABLE 1**

The system 10 can also use the speed/distance model to determine target location in 3-space at a particular time $t$. First, the range $r_{i_t}$ to the target from each sensor $X_i$ at the current time $t$ is $r_{i_t} = \sqrt{(t - T_{R_i})^2 S^2 + R_i^2}$. Each sensor $X_i$ is located at coordinates $(x_i, y_i, z_i)$. The system 10 used the sensor's location and target range $r_{i_t}$ to generate the surface of a sphere that represents all of the possible locations $(x_t, y_t, z_t)$ of the target at time $t$, the sphere defined by the equation $r_{i_t}^2 = (x_t - x_i)^2 + (y_t - y_i)^2 + (z_t - z_i)^2$. A sphere may be determined for each of the three sensors for a given time $t$. Accordingly, the target location at time $t$ is simply the intersection of three spheres. The system 10 computes this intersection by simultaneously solving the three sphere equations for the three unknown variables $(x_t, y_t, z_t)$.

Due to possible measurement errors that result in no intersection or a non-unique intersection, the system 10 computes the target location $(x_t, y_t, z_t)$ that minimizes the distance from the surface of the spheres of the sensors $X_i$. The system 10 chooses an arbitrary starting point, and iteratively improves the target location until iterative change is less than $1 \times 10^{-10}$ meters. For each iteration the next target location is the arithmetic mean location of the current target location projected onto each sphere. It is noted that the projection of $(x, y, z)$ onto the sphere for sensor $X_i$ is $p_i = (x_{p_i}, y_{p_i}, z_{p_i})$ where:

$$x_{p_i} = r_i \frac{x - x_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}} + x_i$$
$$y_{p_i} = r_i \frac{y - y_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}} + y_i$$
$$z_{p_i} = r_i \frac{z - z_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}} + z_i$$

The system 10 then chooses two or more times to compute the target's location. In the golf ball 42 embodiment, the system selects two times that are the minimum and maximum values for $T_{R_i}$. For the data shown in FIG. 1, these times are 0.8285 and 0.9365 seconds. The ball's computed location at each of these times is $(-0.0884,-0.1075,0.5810)$ and $(0.4172,-0.1338,0.7628)$. Noting the point-to-point difference and timing data, the target object speed is computed to be 49.8 meters per second with a launch angle (angle from horizontal) of 19.75 degrees and side angle (left/right deviation from the target line) of 2.98 degrees left.

Variations of the above-described embodiments are contemplated and readily appreciable to one skilled in the art. For example, there are several different methods for determining the speed of the target. In one alternative method, the speed is determined from the distance and time differences from the first to the last computed positions for the target. In another embodiment, a speed versus time curve is determined by computing the distance and time differences between consecutive pairs of computed positions for the target.

In addition, the accuracy of the system 10 can be improved by detecting the target using more than one instance of the system 10 or with more than three sensors. When employing multiple

systems 10 over the same period, the computed values would be averaged over the set of devices. When employing multiple systems over different periods, the extrapolated trajectory parameters may be more accurate. It is also desirable to determine the spin axis and spin rate of the object. Using the concrete example of a golf ball, the problem is to modify the golf ball in such a way that speed, direction, spin and spin axis angle can be remotely measured without perceptively changing the appearance or apparent properties (surface finish, mass, elasticity, magnetic properties, etc.). To implement remote sensing of speed, direction, spin and spin axis angle of an object, the sensing device ideally sees (1) the target object and (2) the rotation of the target object.

In order to enable a radar sensor 110 to view the rotation of a target object, one embodiment places a contrasting marking material beneath the visually opaque surface of the target object (*e.g.*, beneath the outer layer of a golf ball 42 or other object such as a baseball). In another embodiment, a contrasting portion that represents a pre-existing contrasting region of the object is observed. The description that follows refers only to the use of a contrasting marking material although it is understood that a contrasting portion of the object could also be considered. Then a remote sensing device, such as radar sensor 110 is used to sense the contrasting marking material by using an electromagnetic wavelength that passes through the visually opaque surface material. As the object spins, the sensor detects signal variations due to the contrasting material spinning in and out of radar beam view.

One advantage of this embodiment is that the modification of the target object is not visible to a user, such as a golfer 44. As relates to golf balls, they are generally manufactured with a visually

opaque cover material that is transparent to K-band, X-band, and Ka-band radar emissions of the sensors 110, 112, and 114 of the present invention. The contrasting material placed beneath the cover of a golf ball is either more or less reflective than the golf ball's core material to K-band, X-band, and Ka-band radar emissions. It is important that ball modification does not alter the mass or performance characteristics of the ball. Accordingly, the material should be thin, highly reflective (or having a substantially different reflectivity than the ball's core), have a density that is approximately the same as the cover material of the ball, and have a small surface area relative to the surface area of the ball's core.

It is noted that the more radar reflective regions under the surface of a target object may be aid the determination of the speed/direction parameters as described above. In particular, the sensors 110, 112, and 114 may receive a stronger amplitude signal over a wider range of their respective beams when the target object includes one or more such reflective contrast regions. This is particularly true for a target object that is not highly reflective such as a baseball for example. Accordingly, the contrast region configurations described below may also be used to improve the determination of the speed/distance parameters for a target object.

In one embodiment of the invention one or more pieces of aluminum tape are placed on the surface of a golf ball's core beneath the cover material. The aluminum tape acts as a contrasting material that is highly reflective for K-band, X-band, and Ka-band sensors, much more so than the solid core of the golf ball. In one embodiment the aluminum foil tape is an off the self product of Tyco Adhesives™ of Norwood, Massachusetts, in particular "Nashua 322." The Nashua 322 aluminum foil tape has a thickness of 0.05 millimeters and the adhesive is rubber-

based.

Several different configurations of contrast material placement are possible in this embodiment where the diameter of the core 312 is about 1.51 inches, such as shown FIG. 11A to FIG. 14B. As shown in FIG. 11A and FIG. 11B, the contrast material 314 may include two round dots each .75 inches in diameter of aluminum at opposite poles of the ball 42 where the contrast material 314 is placed on the core 312 of the ball 42 and under the visually opaque surface 310 of the ball 42. In another embodiment shown in FIG. 12A and 12B, the contrast material 314 includes one round dot with .75-inch diameter of aluminum foil. The contrast material diameter may also have different size diameters including one inch, for example. FIG. 13 depicts another contrast material configuration that includes four .25 inch round dots 314 placed on the vertices of a tetrahedron. FIG. 14A and FIG. 14B depicts another configuration of contrast material 314 where the round dots have different sizes, in particular, .75 inches and .25 inches in diameter with centers approximately 110 degrees apart.

FIG. 10 is flowchart of an exemplary method of determining the spin rate and spin angle for target modified as described above. In brief, at step 302, the mark times of the spinning object 42 are identified. As the target object spins, the contrasting markings spin into and out of radar beam of the sensor. When in the beam, the highly reflective markings cause a temporary change in the amplitude of the signal received by the sensor. By noting the times of the amplitude changes, the system 10 can determine when the markings appear at each sensor. In step 304, the direction vector of the target object is identified. Using the computed trajectory of the object (location, speed and direction) as described, the system 10 can compute a direction vector from the sensor

to the object that is associated with each of the marking appearances identified at step 302.

At step 306 the spin rate of the target object 42 is determined based on the identified mark times and direction vectors. The identified mark times are interpreted in light of how the object is marked with contrast material. For example, assume the object is marked with one round dot. System 10 computes spin rate by first computing the angle of rotation between two mark observations via the direction vectors. Because the ball has is moving past the sensor as it rotates, the computed angle is not based on a complete revolution. The spin rate is then determined from the angle of rotation and the elapsed time between mark observations. It is noted that multiple pairs of mark observations from multiple sensors may be used to reduce any spin rate measurement error. Note also that the target object can have multiple contrast markings to provide multiple observations per revolution of the target. Using multiple contrast markings thus increases the number of measurements made during each revolution of the target object and reduces the time required to obverse multiple marks and thus determine the spin rate.

At step 308, the spin angle of the target object is computed. In the present invention, the system 10 creates a line of longitude that indicates a great semi-circle on the sphere of the object that intersects the spin axis twice. By noting that object spin causes any point on the line of longitude to move perpendicular to that line of longitude, the spin angle is computed by first identifying a pair of mark observations for the same contrast mark during one revolution as seen by two different radar sensors. It is noted that when these two mark observations occur, the mark's line of longitude is also passing a line between the center of the ball and the sensor, *i.e.*, the direction vector. Then the points on the target object where direction vector is pierced are computed. The

position of the line of longitude of the mark at two different times is computed based on the spin rate and elapsed time between the mark observations. Based on these calculations, the spin axis is defined as the intersection of two planes defined by the two positions of the line of longitude.

FIG. 15 is a flowchart of a preferred method for identifying marks times 302. In the first step 322, the target signal amplitude is computed over time. In detail, the system 10 transforms the time domain, signal data to the frequency domain using a maximum entropy method as described by Press, William H., and et al. *Numerical Recipes in C.* Cambridge University Press. 1992, which is incorporated by reference for its teachings on maximum entropy algorithms. The system 10 converts the time domain voltage signal to the frequency domain at intervals of approximately 3.6 microseconds. The system 10 uses 81 samples centered at the time of interest, with the number of poles for the maximum entropy method set to 75 to generate the frequency domain representation of the signal. System 10 computes the sum of the amplitude values at 100 evenly spaced frequency values for data having a frequency range of plus and minus 500 Hertz based on the frequency domain information derived for the speed/distance as described above.

At step 324, the method 302 identifies the locally minimum points on the amplitude curve determined at step 322. In detail, system 10 locates amplitudes that are lower than both the samples at the prior and subsequent times (time adjacent samples). The system 10 generates the curve of the located amplitudes by using linear interpolation between minima samples. It is noted that the generated curve represents the signal strength generated by the radar sensor when the radar signal reflects off areas of the target without contrast markings.

At step 326, the identified minimum points are adjusted. In particular, they are normalized with respect to the generated minima curve. They can be normalized according to the following equation:

$$a = \begin{cases} \dfrac{(a-m)\left(\dfrac{a}{m}-1\right)}{100} - 10 & \text{if } \dfrac{(a-m)\left(\dfrac{a}{m}-1\right)}{100} - 10 > 0 \\ 0 & \text{if } \dfrac{(a-m)\left(\dfrac{a}{m}-1\right)}{100} - 10 \le 0 \end{cases}$$

where $a$ is the amplitude and $m$ is the value of the minima curve. The $(a-m)$ term represents the signal without the contribution of the ball to the signal. The $\left(\dfrac{a}{m}-1\right)$ term represents the ratio of the brightness (radar signal sensitivity) of the contrast marking as compared to the core of the ball 42. The other constants in the equation are used to scale the amplitude values and eliminate relatively slight amplitude peaks that may be spurious.

At step 328, the amplitude curve peak times are identified. It is noted that there is an amplitude peak associated with each stretch of non-zero amplitude values in the adjusted amplitude curve. The system 10 identifies the maximum value for each non-zero sequence. The time of the amplitude peak is the center of the time range defined by linearly interpolating the half-height amplitude peaks. Using the computed trajectory of the object (location, speed and direction) as described, the system 10 computes a direction vector from the sensor to the object that is associated with each of the marking appearances identified at step 328.

As noted at step 306 the spin rate is computed. In detail, for each consecutive pair of amplitude

peaks for each sensor, the system 10 computes a value for the spin rate. The system 10 then computes the median of the spin rate measurements. Then any measurements that differ from the median spin rates value by more than ten percent are eliminated. The mean of the remaining measurements is computed as the final spin rate. In further detail, the system 10 computes the angle $\alpha$ between the each set of direction vectors in units of radians. The computed angle is the differential from a full rotation of the object due to the movement of the object past the sensor. Other contrast marking configurations (such as shown in FIG. 11A to FIG. 14B) may enable the system 10 to observe more than one full revolution between appearances of the same marker. It is noted that the spin rate is computed in units of radians per second implied by amplitude peaks at times $t_0$ and $t_1$ in units of seconds based on the equation: spin rate $= \dfrac{2\pi - \alpha}{t_1 - t_0}$.

At step 308, the spin angle of the target object is computed. In brief, the system 10 identifies pairs of amplitude peaks that are generated by same contrast marking on the same revolution as measured by two different sensors and computes the spin angle from each pair. Any angle measurements that differ from the median of all of the angle measurements by more than ten percent are eliminated. The final spin angle is then computed as the mean of the non-eliminated measurements. The computation of a spin angle implied by a pair of amplitude peak measurements is explained in detail with reference to FIG. 16 and FIG. 17. FIG. 16 is a flowchart of an exemplary method for determining a spin angle from amplitude peak measurements. FIG. 17 is a diagram of the geometry associated with determining a spin angle based on two amplitude peak measurements.

In first step of the method 308 for determining the spin angle (step 332), the system 10 locates the intersection of the two direction vectors with the surface of a unit sphere centered at the origin. A computed point measured at time $t_0$ as $P_0$ and the point measured at time $t_1$ as $P_1$ shown in FIG. 17 do not represent the position of the marking at times $t_0$ and $t_1$, but rather to points on a line of longitude containing the marking at those times.

At step 334, the system 10 computes the location of a second point $P_1'$ on the line of longitude at time $t_0$. By definition, the point $P_1$ on the line of longitude moves perpendicular to the line of longitude as the ball spins. Accordingly, the system 10 computes point $P_1'$ such that points $P_0$, $P_1$ and $P_1'$ on the surface of the unit sphere form a right triangle where the hypotenuse of the triangle is a segment connecting $P_0$ and $P_1$. The system 10 computes the length of this segment as the Euclidean distance between $P_0$ and $P_1$ the distance between $P_1$ and $P_1'$ from the spin rate and the elapsed time from $t_0$ to $t_1$. At step 334, the system 10 computes a second point $P_0'$ on the line of longitude at time $t_1$ used the process for computing $P_1'$.

At step 338, the system 10 computes the spin axis by constructing and intersecting two planes. One plane contains the origin and points $P_0$ and $P_1'$, the other plane contains the origin and points $P_0'$ and $P_1$. The line of intersection of these two planes is the spin axis. At step 340, the system 10 determines the spin axis as a spin angle relative to the ball's initial velocity vector.

Accordingly, the spin axis determination is then not dependent on the exact orientation of any coordinate system.

While this invention has been described in terms of a best mode for achieving this invention's objectives, it will be appreciated by those skilled in the art that variations may be accomplished in view of these teachings without deviating from the spirit or scope of the present invention. For example, the present invention may be implemented using any combination of computer programming software, firmware or hardware (*e.g.*, a software language other than Java, such as C++ or others may be used to implement the invention). As a preparatory step to practicing the invention or constructing an apparatus according to the invention, the computer programming code (whether software or firmware) according to the invention will typically be stored in one or more machine readable storage mediums such as fixed (hard) drives, diskettes, optical disks, magnetic tape, semiconductor memories such as ROMs, PROMs, etc., thereby making an article of manufacture in accordance with the invention. The article of manufacture containing the computer programming code is used by either executing the code directly from the storage device, by copying the code from the storage device into another storage device such as a hard disk, RAM, etc. or by transmitting the code on a network for remote execution.